EXACT ASYMPTOTICS IN EIGENPROBLEMS FOR FRACTIONAL PROCESSES

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Résumé

Eigenproblems frequently arise in theory and applications of stochastic processes, but only a few have explicit solutions. Those which do, are usually solved by reduction to the generalized Sturm-Liouville theory for differential operators. This includes the Brownian motion and a whole class of processes, derived from it by means of linear transformations. The more general eigenproblem for the fractional Brownian motion is not solvable in closed form, but the exact asymptotics of its eigenvalues and eigenfunctions can be obtained, using a method based on analytic properties of the Laplace transform. We set up a framework for the spectral analysis of the fractional type covariance operators, corresponding to an important family of processes, which includes the fractional Brownian motion, its noise, fractional Ornstein-Uhlenbeck process and the integrated fractional Brownian motion. We obtain accurate asymptotic approximations for the eigenvalues and the eigenfunctions and consider their applications to parameter estimation and filtering.

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